ASTRON 329/429, Fall 2017 – Problem Set 1

Due on Oct. 5, in class.

Solve problems 3.2, 3.3, 3.4, and 3.5 in Ryden, plus the following problems:

I. Hubble's law. In the real Universe, the expansion is not completely uniform. Rather, galaxies exhibit some random motion relative to the overall Hubble expansion, known as their *peculiar velocity* and caused by the gravitational pull of their near neighbors. Supposing that a typical (i.e., root mean square) galaxy peculiar velocity is 600 km/s, how far away would a galaxy have to be before it could be used to determine the Hubble constant to 10 percent accuracy, supposing,

a) that the true value of the Hubble constant is 70 km/s/Mpc?

b) that the true value of the Hubble constant is 1,000 km/s/Mpc?

Assume in your calculations that the galaxy distance and redshift can be measured exactly (which is not true in real observations!) and that you can only measure these quantities for a single galaxy.

II. Planck spectrum. Starting from the Planck energy spectrum

$$\epsilon(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{\exp\left(h\nu/k_{\mathrm{B}}T\right)-1}, \qquad (1)$$

a) Show that $\epsilon(\nu)$ peaks for $\nu \approx 2.8k_{\mathrm{B}}T/h$. In your derivation, you may use a computer program such as Mathematica to numerically evaluate the root of a dimensionless equation.

Using the fact that the CMB temperature is $T = 2.75$ K, derive the peak frequency and corresponding wavelength. Thus, confirm that the CMB peaks in the microwaves.

b) Derive an expression for the number density of photons (number of photons per unit volume) as a function of temperature. What is the number density of CMB photons at the present time?

III. Acceleration equation. Starting from the Friedmann and fluid equations, derive the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \qquad (2)$$

showing all steps. Notice how the curvature k cancels out.

IV. Different form of the RW metric. In class, we stated that the spatial part of the Robertson-Walker metric, for $a = 1$, can be written as

$$ds_3^2 = \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \qquad (3)$$

where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$, and $k = 0$ in flat space, $k > 0$ for a closed universe, and $k < 0$ for an open universe.

Ryden expresses the RW metric in a different form. To connect the class discussion to the textbook, show that for a suitable change of coordinates the RW metric can also be written as

$$ds_3^2 = dr^2 + S_\kappa(r)^2 d\Omega^2, \tag{4}$$

where

$$S_\kappa(r) = \begin{cases} R_0 \sin\left(r/R_0\right) & (\kappa = +1;\ \text{closed/spherical}) \\ r & (\kappa = 0;\ \text{flat/Euclidean}) \\ R_0 \sinh\left(r/R_0\right) & (\kappa = -1;\ \text{open/hyperbolic}) \end{cases} .$$

In this equation, the radius R_0 is a constant characterizing the radius of curvature and the dimensionless $\kappa \in \{0, \pm 1\}$.